Numeric Dispersion Analysis of 3D Envelope-Finite Element (EVFE) Method

Weijun Yao and Yuanxun Wang
Department of Electrical Engineering, University of California at Los Angeles,
405 Hilgard Avenue, Los Angeles, CA90095-1594, U.S.

Abstract—In this paper, the numeric dispersion error of 3D envelope-finite element (EVFE) is analyzed with several examples. The results show that the EVFE method has a much lower dispersion error compared to the unconditionally stable finite element time domain (FETD) method. Furthermore, in the analysis of high Q structures such as resonators or filters, the EVFE technique can obtain the resonator frequency or frequency response not only efficiently but also accurately.

I. INTRODUCTION

By properly setting the parameters in Newmark-Beta formulations, one can easily obtain unconditional stability in finite element time domain (FETD) method [1][2]. This method makes time step in the FETD method no longer governed by the spatial discretization of the mesh rather by the spectrum of the excitation signal, such that the time step can be very large while the algorithm is still stable. However, in modern optical or wireless communications, the base band signal is usually modulated to a very high carrier frequency, so that a very small time step is required in FETD in order to follow the fast varying carrier. As a result, the numeric dispersion error can be huge as the time step exceeds restriction by the Nyquist sampling criterion.

Envelope finite element technique (EVFE) can overcome the above shortcomings while keeping the advantage of unconditional stability [3][4]. By de-embedding the carrier frequency from the excitation signal, only the base band complex signal envelope is simulated. Thus, the time step is only restricted by the base band signal's bandwidth, which is usually much smaller than the modulated signal's highest frequency. In this sense, given the same time step in EVFE technique and unconditionally stable FETD method, EVFE could have a much lower dispersion error. From the point of view of signal and system theory, the FETD algorithm is actually a low pass system, while the EVFE algorithm is more like a band pass system, which is more suitable to process the modulated signals.

II. 3D EVFE FORMULATIONS

A simple derivation of the 3D EVFE formulations will be presented in this section [3][4]. From Maxwell equations, the vector wave equation can be easily obtained:

$$\nabla\times\left(\frac{1}{\mu}\nabla\times\vec{E}\right)-\omega^2\varepsilon\vec{E}=-j\omega\frac{1}{\mu}\vec{J} \qquad (1)$$

Based on the vector finite element method, we can recast equation (1) into the following form:

$$-\omega^2 e_j\mathbf{Q}+e_j\mathbf{P}=-j\omega\int_v\frac{1}{\mu}\overrightarrow{N_i}\bullet\vec{J}dv \qquad (2)$$

0-7803-7846-6/03/$17.00 ©2003 IEEE

Where
$$\mathbf{Q} = \int_v \varepsilon N^j N^i dv$$
$$\mathbf{P} = \int_v \frac{1}{\mu} (\nabla \times \vec{N}^j)(\nabla \times \vec{N}^j) dv \tag{3}$$

Define signal envelopes as:

$$e_j(t) = u_j(t) e^{j\omega_c t}$$
$$J_z(t) = j_z(t) e^{j\omega_c t} \tag{4}$$

Where ω_c is the carrier frequency and *u, j* are the complex envelopes, setting ω_c to zero leads to the FETD case. The relationship between the frequency domain field and time domain field envelope are:

$$j\omega \rightarrow \frac{\partial}{\partial t} e^{j\omega_c t} + j\omega_c e^{j\omega_c t} \tag{5}$$
$$-\omega^2 \rightarrow \frac{\partial}{\partial t^2} e^{j\omega_c t} + 2j\omega_c \frac{\partial}{\partial t} e^{j\omega_c t} - \omega_c^2 e^{j\omega_c t}$$

Using relationship (5), the time domain envelope equation can be obtained from (2):

$$\mathbf{T}_1 \frac{d^2 u_j}{dt^2} + \mathbf{T}_2 \frac{du_j}{dt} + \mathbf{T}_3 u_j = -\left(\frac{\partial \mathbf{f}}{\partial t} + j\omega_c \mathbf{f}\right) \tag{6}$$

Where $\mathbf{T_1}$, $\mathbf{T_2}$ and $\mathbf{T_3}$ are complex matrixes and **f** is a complex vector.
Using Newmark-Beta formulation to discretize equation (6) in the time domain [3], the complex signal envelope u_j can be solved.

III NUMERICAL RESULTS AND DISPERSION ERROR ANALYSIS

Two numerical models will be used to analyze the dispersion error of the EVFE technique. Furthermore, the results will be compared to those from the unconditionally stable FETD method to show the elegance of the EVFE technique. The first model is 3D free space model (see Fig.1). The source generates a modulated Gaussian pulse with the center frequency 2GHz and bandwidth of 1GHz. The observation point is 22.5mm away from the source.

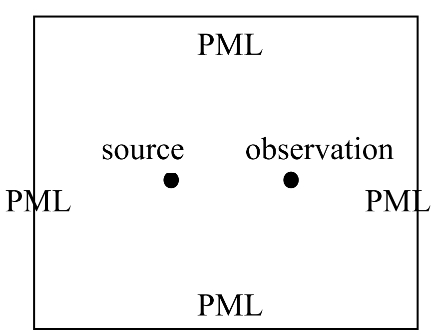

Fig.1 3D free space model

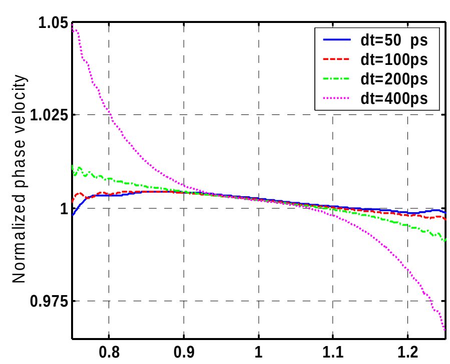

Fig.2 Normalized phase velocity versus different time steps

The phase velocity will be calculated in this model in order to see the dispersion error. Fig.2 shows the normalized phase velocity versus different time steps. The results indicate that, with the EVFE technique, the dispersion error is very small even if the time step becomes very large. When the time step is 400ps, which is about 28 times as big as that restricted by the Courant-Friedrich-Levy (CFL) condition, the phase velocity error is less than 5%. As the time step size increases, the dispersion error also increases. However, in the region close to the normalized frequency 1, one can obtain lowest dispersion error.

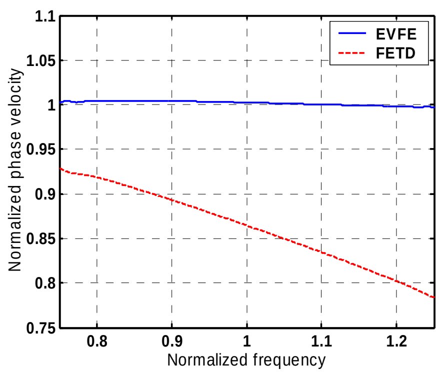

Fig.3 phase velocity in EVFE and FETD methods

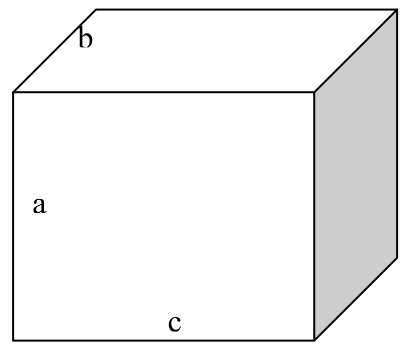

Fig. 4 waveguide resonator model

Fig.3 shows the results of phase velocity using both EVFE and FETD methods under the condition of time step size equals to 100ps. Inside the excitation bandwidth, the phase velocity calculated by EVFE technique has a very small error (about 0.5%). On the other hand, the result from FETD method has a huge error (about 20%) at the highest frequency, and the error dramatically increases as the frequency increases.

The second example is a waveguide resonator filled with air (see fig.4). The dimensions are: a=b=c=100mm, and the theoretical resonator frequency forTE_{101} mode is fr=2.121GHz. To obtain the resonator frequency numerically, we use a modulated Gaussian pulse with the center frequency at 1.5GHz and bandwidth of about 2GHz to excite this structure. Fourier transformation of the response signal is then employed to obtain the resonator frequency. In Fig. 5, we plot the error of the resonator frequency with both EVFE and FETD methods versus the normalized time step size (normalized to the time step size dt0 restricted by CFL condition). The error is defined by:

$$error = \frac{\left| f_{calculated} - f_r \right|}{f_r} \times 100\% \qquad (7)$$

Inside the excitation bandwidth, the EVFE's dispersion error is much smaller than the FETD's error. As the time step increases, the FETD's error increases quickly while EVFE's error increase much more slowly. In Fig.6, the errors versus the different carrier frequency (normalized to the theoretic resonator frequency) are also plotted under the same time step size dt=8*dt0. The EVFE's error is symmetric according to the normalized frequency 1 while the FETD's error is almost constant for different carrier frequencies. When the carrier frequency is close to normalized frequency 1, the EVFE has a minimum error close to 0. However, when the carrier frequency is larger than 1.5, FETD method can not obtain the valid result. This is because the FETD method is a low pass type of algorithm, and high frequency modulated signal will be filtered out.

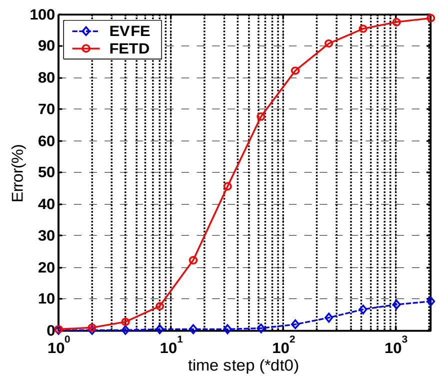

Fig.5 error versus the normalized time step size

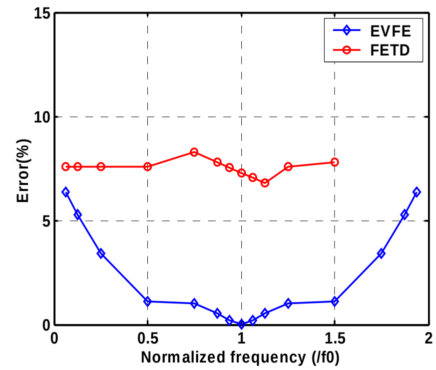

Fig.6 error versus the different carrier frequency

IV CONCLUSION

The numeric dispersion error of EVFE technique is analyzed in this paper. The results show the EVFE has a much lower dispersion error compared with the FETD method because the EVFE method is a band pass algorithm while the FETD is a low pass algorithm. Therefore, the EVFE is a more efficient and precise technique in the analysis of high Q structures such as resonators or filters.

REFERENCES

[1] J-F. Lee, R. Lee and A. Cangellaris, "Time-domain finite-element methods", *IEEE Trans. Antennas and Propagation,*Vol. 45, NO.3, pp. 430-442, March 1997.

[2] S. D. Gedney and U. Navsariwala, "An unconditionally stable finite element time-domain solution of the vector wave equation", *IEEE Microwave and Guided Wave Letters*, vol. 5, NO. 10, pp. 332-334, Oct. 1995.

[3] Y. Wang and T. Itoh, "Envelope-finite element (EVFE) technique – a more efficient time domain scheme", *IEEE Trans. Microwave Theory Tech.* Vol.49, pp2241 -2247, Dec. 2001.

[4] W. Yao and Y. Wang, "Envelope - finite element (EVFE) technique for Electromagnetics in general media", submitted to *IEEE Trans. Antenna and Propagation.*

[5] T. Namiki and K. Ito, "Investigation of numerical errors of the two-dimensional ADI-FDTD method", *IEEE Trans. on Microwave Theory and Techniques.* Vol. 48, NO.11, pp 1950-1956, Nov. 2000.

[6] T. Namiki and K. Ito, "Numerical simulation of microstrip resonators and filters using the ADI-FDTD method", *IEEE Trans. on Microwave Theory and Techniques.* Vol. 49, NO.4, pp 665-670, April 2000.

[7] H.-P. Tsai, Y. Wang and T. Itoh, "An unconditionally stable extended (USE) finite element time domain solution of active nonlinear microwave circuits using perfectly matched layers", *IEEE Trans. Microwave Theory Tech.* Vol.50, pp2226 -2232, Oct. 2002.