Jacobi-Davidson-Type Algorithms with Interior Multigrid-Scheme for the Simulation of Electromagnetic Resonator Structures with Gyromagnetic Materials

M. Clemens*, S. Feigh, R. Schuhmann, T. Weiland
Technische Universität Darmstadt, Germany, Dept. of Electrical Engineering and Information Technology, Computational Electromagnetics Laboratory

In some electromagnetic resonators applications ferrite materials are used, e.g. in the construction of particle accelerator cavities, for the purpose of frequency tuning. Such gyromagnetic material properties are mathematically described with the complex-valued Polder tensor $\vec{\vec{\mu}}$, which is non-diagonal and non-symmetric. The eigenvalue problem for the resonant fields of such structures $\nabla \times (\vec{\vec{\mu}}^{-1}(\omega)\nabla \times \vec{E}) = \omega^2 \vec{\vec{\varepsilon}}(\omega)\vec{E}$ can be discretized with the Finite Integration Technique (FIT) (T. Weiland, *Int. J. Num. Modelling*, 9, 295-319,1996), a technique closely related to that of lowest order Whitney Finite Elements (WFEM), to yield an algebraic eigenvalue problem

$$[\mathbf{M}_{\vec{\vec{\varepsilon}}}^{-1}(\omega_{est})\tilde{\mathbf{C}}\mathbf{M}_{\vec{\vec{\mu}}^{-1}}(\omega_{est})\mathbf{C} - \omega^2\mathbf{I}]\,\hat{\mathbf{e}} = 0 \quad \Leftrightarrow \quad \mathbf{A}\,\mathbf{x} = \lambda\,\mathbf{x}. \qquad (1)$$

The frequency dependency of the materials is neglected with an estimated frequency $\omega_{est.}$ The nonzero eigenpairs with smallest real part of the non-symmetric and complex-valued system matrix $\mathbf{A} = \mathbf{M}_{\vec{\vec{\varepsilon}}}^{-1}\tilde{\mathbf{C}}\mathbf{M}_{\vec{\vec{\mu}}^{-1}}\mathbf{C}$ are of interest. For their calculation the Jacobi-Davidson (JD) subspace iteration algorithm projects the problem (1) onto a successively generated search space, which yields an eigenvalue problem of considerably smaller dimensions. The search space is augmented with the solutions $\mathbf{v}$ of the correction equation (CE) $(\mathbf{I}-\mathbf{q}\mathbf{q}^T)(\mathbf{A}-\lambda\mathbf{I})(\mathbf{I}-\mathbf{q}\mathbf{q}^T)\mathbf{v} = -(\mathbf{A}-\lambda\mathbf{I})\mathbf{q}$ in each step, where $(\mathbf{q},\lambda)$ is the most recent eigenpair approximation. For the approximative solution of this inner linear system several simplifications of the CE itself and several iterative solution schemes such as e.g. the preconditioned BiCGstab algorithm are applicable. A recently developed geometric multigrid (MG) scheme, which relies on the separation of grid incidence and material matrices within the FIT (and certain WFEM) formulations, is found to achieve an improved asymptotical complexity required for large 3D simulations.

In Fig. 1 the convergence history for the 3 lowest eigenvalues of a resonator test structure with a ferrite material insert ($3 \cdot 10^4$ Dof) was analyzed when using the system matrix $\mathbf{A}$ in a simplified CE, in Fig. 2 the multigrid solver shows the fastest convergence.

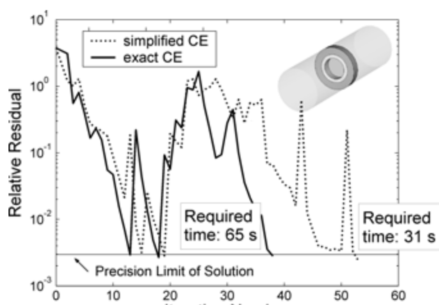

Fig. 1. Convergence history of JD scheme for a cylindrical resonator test structure.

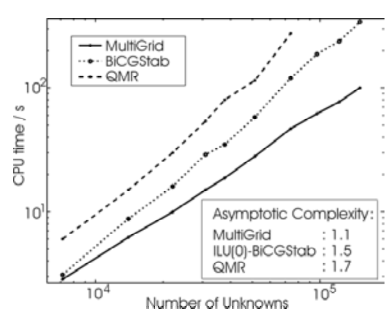

Fig. 2. Asymptotic complexity of CE solution. (Using MATLAB adds 0.1 to the slope.)

0-7803-7846-6/03/$17.00