Perfectly Nested Finite Element Spaces Using Generalized Hanging Variables

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Starting from an initial mesh, discretizations of higher resolution can be constructed very easily by partitioning existing finite elements (FE's) recursively [1]. Besides guaranteeing a lower bound on mesh quality, the resulting nested structure adds a high degree of regularity to the FE discretization, which greatly simplifies the application of fast geometrical multigrid solvers. At the same time, efficient error control requires the mesh size to adjust locally according to the spatial behavior of the electromagnetic fields and thus implies the need for strongly non-uniform mesh refinement.

Traditional red/green schemes [1] connect sub-domains of unequal refinement levels by layers of irregular elements. However, such elements are usually of inferior quality than their parents and must be removed when the mesh is further refined. Also, the resulting sequence of finite element spaces is no longer fully nested, and the construction of inter-grid transfer operators becomes more complicated.

As shown in Fig. 1, our alternative approach allows finite elements of unequal refinement levels to touch. As a result, the FE hierarchy is always perfectly nested. To maintain proper continuity of fields across element interfaces [2], we extend the concept of hanging nodes to H(curl) conforming basis functions of higher order [3]. Specifically, we propose to realize hanging variables by imposing appropriate constraints on fine mesh basis functions. Since the underlying process can be interpreted as a Galerkin type restriction operation, the proposed method is particularly attractive in the context of multigrid solvers. Our investigations have shown that the construction of appropriate constraints is by no means unique. As indicated in Fig. 2 and Table I, the strategy chosen has great influence on the resulting number of variables or non-zero matrix entries, respectively, as well as the numerical properties of the system matrix.

The present talk focuses on the computational efficiency of the generalized hanging variables framework. We first prove that, for the whole set of permissible constraints, the corresponding sequences of finite element spaces remain perfectly nested. We then propose one specific method which is easy to implement and combines low memory requirements and rapid numerical convergence. Numerical examples are given to validate our findings.

REFERENCES:
[1] U. Rüde, *Mathematical and Computational Techniques for Multilevel Adaptive Methods*, Philadelphia, PA: SIAM, 1993.
[2] J.P. Webb, S. McFee, "Nested Tetrahedral Finite Elements for h-Adaption," *IEEE Trans. Magn.*, vol. 35, pp. 1338-1341, 1999.
[3] V. Hill, O. Farle, R. Dyczij-Edlinger, "A Stabilized Multi-Level Vector Finite Element Solver for Time-Harmonic EM Waves", accepted for publication in *IEEE Trans. Magn.*.

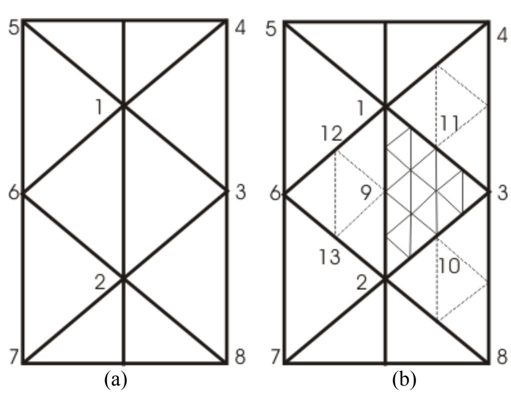

Fig. 1. (a) Initial mesh. (b) Non-uniformely refined mesh.

TABLE I
CONSTRAINTS ON BASIS FUNCTIONS

Strategy	Variables	Non-Zeros	Iterations
A	77350	2 147 185	12
B	58030	1 923 775	147
C	58030	1 563 746	10

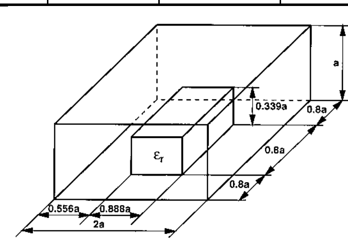

Fig. 2. Dielectric obstacle inside waveguide.

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