Dispersion properties of periodic grounded structures via equivalent network synthesis

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1. INTRODUCTION

The increasing interest on metamaterials and frequency band-gap structures has been motivated by the large number of recently discovered engineering applications in the field of microwave and antennas. Among the great variety of technological solution, periodic surfaces printed in stratified dielectric media are important for their low-cost and realization simplicity. One of the main objective is the realization of artificial magnetic surfaces [1], [2]. For patches or dipoles printed-on or embedded-in stratified structure, the actual purpose may be concerned with the realization of compact antennas, and the suppression of surface waves to reduce coupling and diffraction lobes. From the recent literature also emerge a different application which is concerned with the excitation of leaky waves for the purposes of gain enhancement [3].

In order to control leaky wave excitation and surface wave band-gap, it is necessary to have efficient model of the dispersion properties in the irreducible Brillouin zone. The classical approach based on integral equations leads to the numerical searching of complex zeros of MoM matrix determinant. This method is often very complicated and strongly sensitive to the initial guess, especially when searching for leaky-modes. To speed up the process, a method is suggested here, which on one hand constitutes itself a good approximation of the solution, on the other hand may serve for finding the initial guess to start for pursuing the accurate solution. In any case it gives a physically appealing description of the physics. Although the procedure has been devised here for simple geometries, it exhibits a potential applicability for any structures composed by frequency selective surfaces embedded in stratified dielectric region.

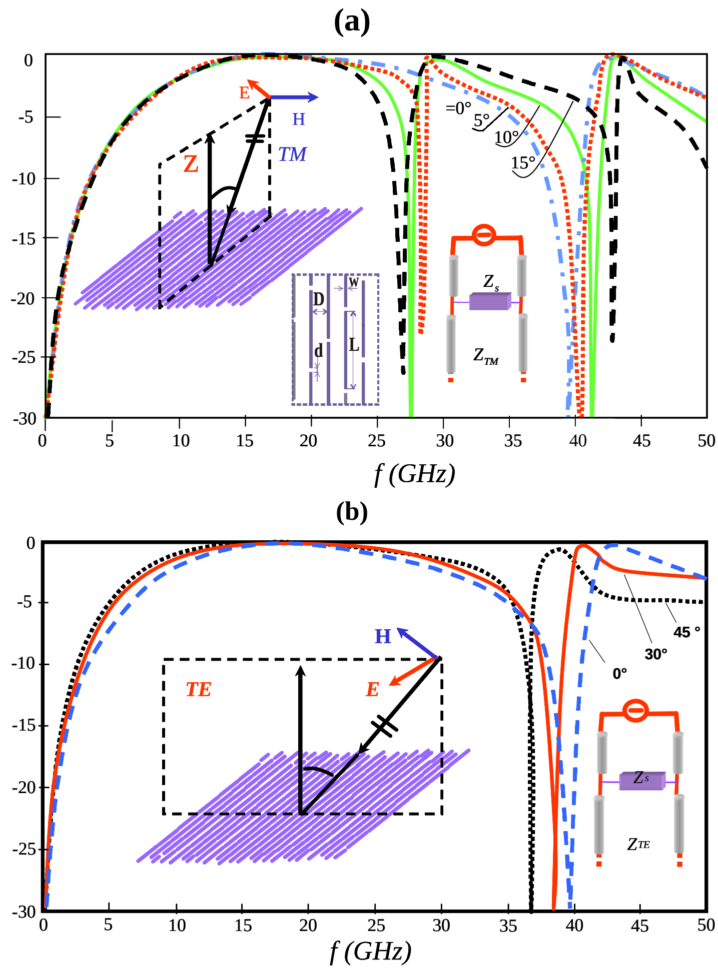

Fig 1 *Reflection coefficient of an FSS (gangbuster "type 2" L= 10 mm, D= 2.51 mm, d= 0.571 mm, w= 0.25 mm.) versus frequency for different scan angles (a) E- plane (TM) and b) H-Plane (TE).*

2. FSS HOMOGENEIZATION AND SELECTION OF POLES AND ZEROS

The approach is based on the model of a free-standing frequency selective surface (FSS) as a basic building block of an equivalent network for describing TE and TM transverse-resonance network. As it is well known, the FSS are realized by compactly interlaced resonant planar elements which leads to a homogenized surface impedance within a certain frequency bandwidth. A large bandwidth synthesis of this

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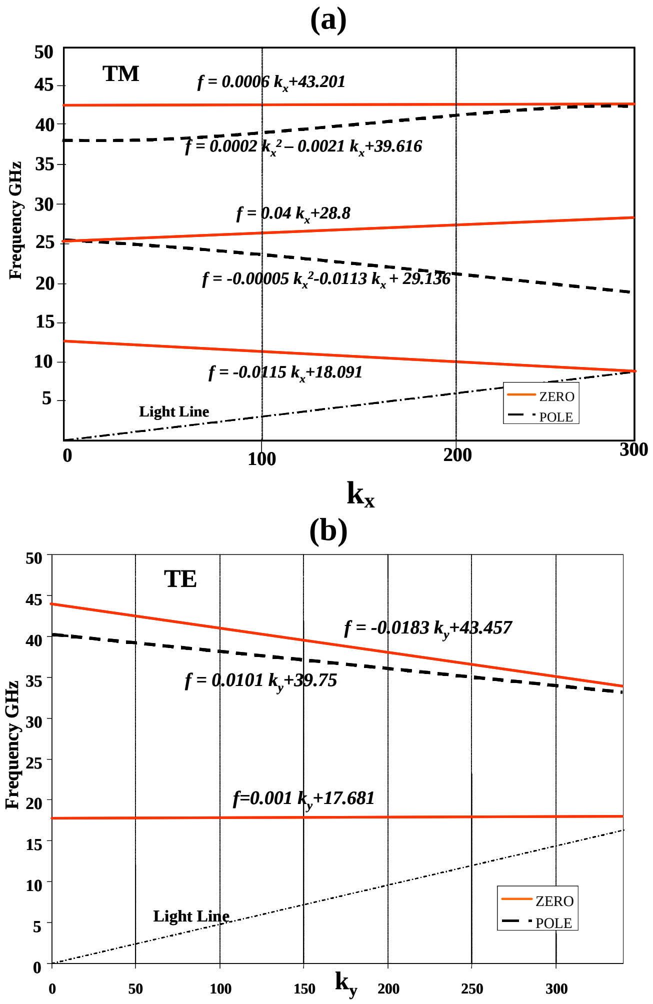


Fig. 2 *Position of zeros and poles of Z_s in the $f - k_x$ plane (TM case figure (a)) and in the f-k_y plane (TE case figure (b)).*

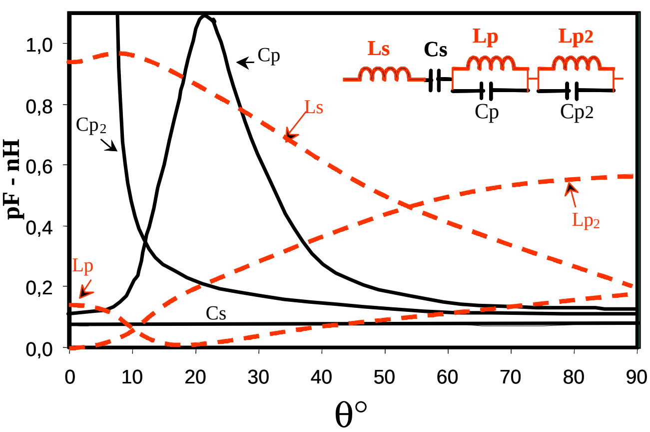


Fig. 3 *Synthesized θ-dependent L an C parameter for the TM case (the circuit for the TE case includes only one paralle-typel block for the same bandwidth)*

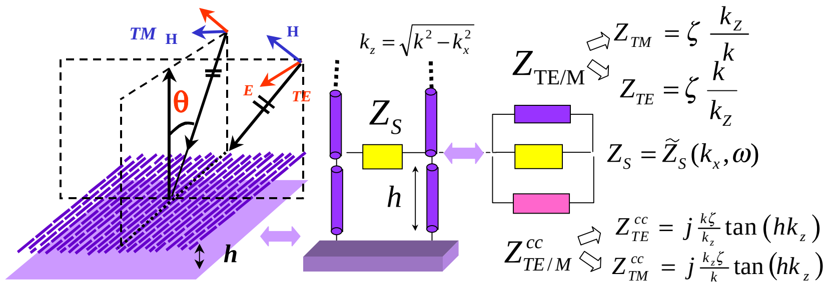
surface through a dispersive (wavenumber dependent) equivalent network is a feasible task also with simple optimization tools. To illustrate the concept, let us consider a simple case of a FSS floating in free space and illuminated by a plane wave. For the sake of simplicity, we consider a "type 2" gangbuster surface [4]. The properties of gangbusters metamaterial have been devised in [5][6]. Fig 1a and 1b shows the TE and TM wide-band behavior of the reflection coefficient, in the E- and H-plane plane, respectively (note that E is always parallel to the dipoles). The results have been obtained by a spectral-domain based Method of Moment (MoM) analysis and next validated by the commercial software presented in [7]. The corresponding equivalent z-transmission line network is shown in the inset of the same figure. An equivalent impedance $Z_s(\theta,\omega)$ (yet undetermined), placed at z=0, simulates in a broadband the response of the FSS surface by account for the local reactive energy associated to the Floquet modes. We make here the assumption that the equivalent impedance is purely reactive due to the assumed absence of losses in the metalization.
From the results of Fig. 1, we note that the response of the FSS is apparently quite variable with the angle of incidence. Actually, this is not so. The characterization of the surface can be described by using few parameters which exhibits weak dispersion characteristics. Indeed, we can neatly individuate poles and zeros of the equivalent impedance $Z_s(\theta,\omega)=\tilde{Z}_s(k_x,\omega)$ (we suppose k_x=kcosθ) because they are placed on the real frequency axes being the impedance purely reactive (note that the plots in Fig. 1 show the reflection coefficient of the z-transmission line; thus, before selecting poles and zeros, the surface reactance must be analytically extracted from the circuit). From the network theory, we know that a reactance LC-function, can be uniquely determined by the positions of poles and zeros and from the behavior at ω=0 and infinity. (This latter condition may be replaced by the value assumed at a point where the response is flat, which can be taken as coincident with the maximum of the investigated bandwidth). The broad-band behavior of these poles (dashed line) and zeros (continuous line) is shown in Fig. 2 as a function of the grazing wavenumber k_x=kcosθ. It is apparent that poles and zeros has a moderate variation with k_x in the Brillouin zone. This suggests a simple strategy for the broad-band description of the FSS in the large k_x-ω domain with the use of a minimized number of parameters.

3. POLE-ZERO INTERPOLATION AND NETWORK SYNTHESIS

The smooth variation of the pole and zero positions versus k_x allows an interpolation of their position in the k_x-ω plane with a first or second order polynomial of k_x (see Fig. 2). This is quite general and it has been verified also when the FSS is embedded in a stratification. By resorting to a pole-zero network synthesis, the impedance function $\tilde{Z}_s(k_x,\omega)$ can be simply obtained. An example of synthesized network is that shown in Fig. 3, where the LC parameters are drawn as a function of θ. Using those values, the original full-wave data have been almost perfectly matched (results are not presented because indistinguishable from the full wave data in the range of drawing of Fig.1). Obtaining the equivalent circuit from the pole-zero position is a straightforward matter, and requires a negligible numerical effort. Thus, the L-C parameters are a consequence of a mathematical procedure, and it is difficult to attribute them a physical meaning. However, a clever choice of the network topology, allows one to physically associate the L-C parameters to the quasi-static capacitance and inductance of the dipoles. For instance, using the network depicted in Fig.3, the dominant C_s and L_s series components (responsible of the zero-pole pairs at lower frequency) can be interpreted as the gap capacitance between the two end-points of contiguous dipoles and the inductance of the dipole, respectively. This also allows to intuitively predict the trend of the elements when moving the FSS geometry.

4. DISPERSION EQUATION

The L-C network in Fig.3 does not give more information than the k_x-dependent full-wave analysis, but allows one to manage an analytical quantity into the dispersion equation. To this end consider a more complicated structure, which consists of the same FSS considered before with a back ground plane at a distance *h*. The outcome is a structure which supports surface and leaky-modes. The z-transmission line description of this structure comprises the same $\tilde{Z}_s(k_x,\omega)$ impedance elaborated before. In general, the presence of the ground plane may modify the equivalent impedance $\tilde{Z}_s(k_x,\omega)$ to be placed in the model; this occurs when higher-order (evanescent) Floquet modes are not significantly attenuated at the ground plane. Thus, the validity of the assumption can be easily verified again for comparison with the full-wave analysis of the total structure (note that the direct synthesis of the grounded structure is more complicated then the Z_s synthesis).



$$\frac{1}{Z^{cc}_{TE/M}} + \frac{1}{Z_{TE/M}} + \frac{1}{Z_s} = 0$$

Fig. 4 *Transverse resonance equation via synthesised network for the grounded FSS*

The transverse resonant equation is now derived from the z-transmission line circuit in Fig. 4. When searching the leaky wave solutions, we assume that the analytical expression of $\tilde{Z}_s(k_x,\omega)$ derived in the pre-elaboration is continued analytically in the complex k_x plane in the region $Re(k_x)<k$ with $\mathrm{Im}(\sqrt{k^2-k_x^2})>0$. The analytical expression of the dispersion equation is next solved without relevant effort by a conjugate gradient method. The same approach is also used for the slow-wave (surface-wave) region. In this case, the polynomial approximations which leads to the dispersivity of Z_s is extrapolated for values of k_x greater that k and less than and less than π/d_x (d_x= periodicity in x), i..e, until the limit of the Brillouin zone.

We note that the approach used here can be applied for both TE and TM modes and for an arbitrary k-direction in the x-y plane. Thus, one can reconstruct with few data arising from the full wave analysis, the dispersion properties of the total Brillouin zone. An example of application is that shown in Fig. 5, where x-propagating TE modes and y-propagating TM modes have been considered (since the particular dipole geometry, the reverse polarizations are not dispersive because don't feel the dipoles). It is interesting to note that around 18,40 GHz, we individuate a leaky mode which has both real and imaginary part very close to zero. This phenomenon occur at the same frequency for both the TE and TM case. When excited by a point

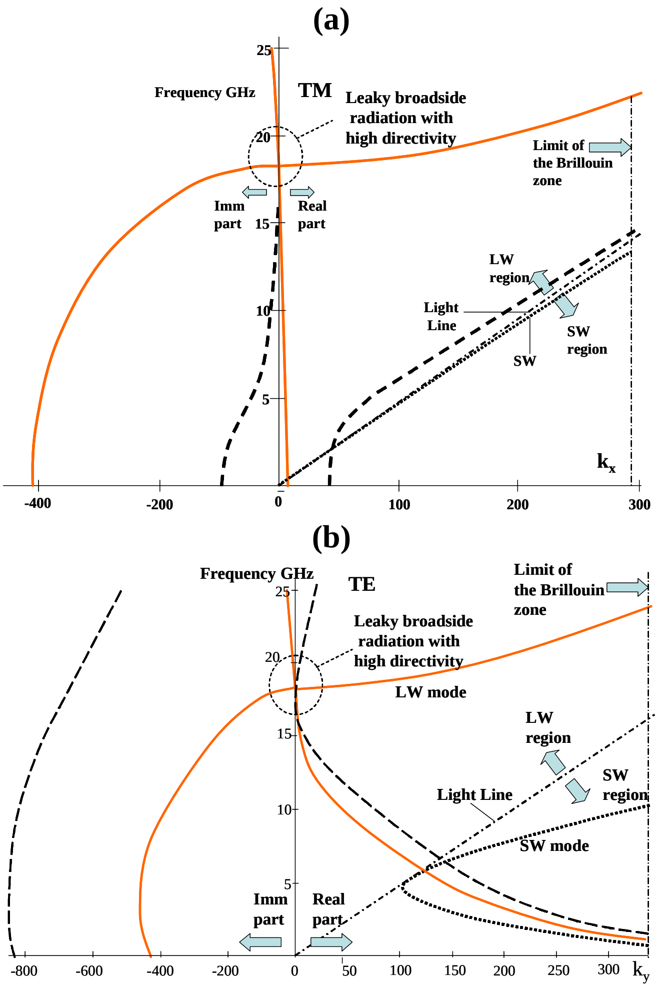


Fig. 5 *Dispersion diagram for leaky and surface wave solution. (a) TM case; (b) TE case*

source, this leaky wave provides a highly directive, perfectly polarized, far-field radiation. This phenomenon has been subject of a dedicated investigation [8].

5. CONCLUSIONS

A method has been presented for the efficient dispersion analysis (associated to both surface-waves and leaky-waves) for printed periodic FSS-type surfaces. The method has been tested for a gangbuster FSS backed by a ground plane. The method consists on a pre-processing performed on the broadband reflection-coefficient data obtained from a full-wave analysis. After that, the FSS is characterized via its resonances for few values of the incident angle. This determines in a straightforward way poles and zeros of an equivalent impedance, simply synthesized by an L-C dispersive circuit. Due to the weak dispersivity of poles and zeros parameters, the L-C circuit can be identified in a simple analytical form which, after analytical continuation, is applied to formulate the transverse resonance equation. The process results much simpler and faster than a full-wave solution for dispersion and provides an accurate initial guess for the iterative identification of the leaky and surface wave solutions.

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