Fast Computing of the first Order Interaction Terms for Scattering Amplitudes in Polarimetric Remote Sensing

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In polarimetric remote sensing of forest, the trunks and branches often are modeled as finite dielectric cylinders and in the resulting multiple scattering problem only the direct terms of single cylinders and the first order interaction terms of pairs are taken into account in computing the far field scattering amplitude. The far field and the scattering amplitude of a finite cylinder can be in a fast way computed by using a truncated infinite cylinder approximation (TICA). I this talk we present a new method how to compute the scattering amplitudes of the interaction terms also in a fast way by using TICA and the Rokhlin translation formula, well-known in the context of the fast multipole methods.

For incident polarization $\hat{\boldsymbol{\alpha}}$ and direction $\hat{\boldsymbol{k}}_i$, the scattering amplitude $f^{V_2,V_1}_{\hat{\boldsymbol{\beta}},\hat{\boldsymbol{\alpha}}}(\hat{\boldsymbol{k}}_s,\hat{\boldsymbol{k}}_i)$ of the first order interaction term of cylinders V_1 and V_2 in the scattering direction $\hat{\boldsymbol{k}}_s$ and for the scattering polarization $\hat{\boldsymbol{\beta}}$ can be given in the terms of the reaction integral of Chiu and Sarabandi as follows,

$$f^{V_2,V_1}_{\hat{\boldsymbol{\beta}},\hat{\boldsymbol{\alpha}}}(\hat{\boldsymbol{k}}_s,\hat{\boldsymbol{k}}_i) = \frac{i\omega\mu_0}{4\pi}\int \boldsymbol{E}_{\boldsymbol{J}_1}(\boldsymbol{r})\cdot \boldsymbol{J}_2(\boldsymbol{r})d\boldsymbol{r} \tag{1}$$

where $\boldsymbol{J}_1$ is the volume current in V_1 induced by the incident plane wave $\hat{\boldsymbol{\alpha}}e^{ik_0\hat{\boldsymbol{k}}_i\cdot\boldsymbol{r}}$ when V_2 is not present, $\boldsymbol{J}_2$ is the volume current in V_2 induced by the incident plane wave $\hat{\boldsymbol{\beta}}e^{-ik_0\hat{\boldsymbol{k}}_s\cdot\boldsymbol{r}}$ when V_1 is not present, and $\boldsymbol{E}_{\boldsymbol{J}}(\boldsymbol{r})$ is the electric field in vacuum due to a source current $\boldsymbol{J}$. The currents $\boldsymbol{J}_1$ and $\boldsymbol{J}_2$ can be computed by TICA in a fast way, but the integral in (1) is essentially 6-dimensional and usually slow to compute directly.

For a fast computing of this integral, we chop the cylinders V_j and the currents $\boldsymbol{J}_j$ into short blocks $V_{j,m}, \boldsymbol{J}_{j,m}$, $m = 1,\ldots,M_j$, $j = 1,2$, and the integral in (1) gets the form

$$\sum_{n=1}^{M_1}\sum_{m=1}^{M_2}\int \boldsymbol{E}_{\boldsymbol{J}_{1,n}}(\boldsymbol{r})\cdot\boldsymbol{J}_{2,m}(\boldsymbol{r})d\boldsymbol{r}.$$

Next the fields $\boldsymbol{E}_{\boldsymbol{J}_{1,n}}$ are expanded about $V_{2,m}$ in plane waves by using the Rokhlin formula, and the resulting integrals are again efficiently computed by using TICA.

In this talk we show how this procedure can be organized into a fast algorithm which greatly reduces the cost of computing the integral in (1). Also numerical examples are presented.