THE KENNAUGH'S CONCEPT OF THE INVERSION POINT INSIDE THE POINCARE SPHERE MODEL OF THE SCATTERING MATRIX AND FURTHER DEVELOPMENTS OF HIS IDEAS

Zbigniew H. Czyz
Telecommunications Research Institute, Warsaw Poland

INTRODUCTION. Edward Morton Kennaugh (1920-1983), in his last paper (short communication) published in 1981 [1], recalled his geometrical interpretation of polarization dependence of the radar crossection (RCS). That dependence has been discovered by him and announced for the first time in his two OSU Antenna Laboratory Reports of the years 1950 and 1951 [2], [3]. Kennaugh noticed that his results '*described 30 years earlier seemed to have been generally overlooked*'. Now we can observe that those results, such simple and so important for practical applications, have been '*generally overlooked*' also during the next 20 years. The aim of this paper is to recall his original '*inversion point*' concept as a corner-stone of the Poincaré sphere model of the scattering matrix and to present some further developments of his ideas based on that concept.

KENNAUGH'S MODEL OF THE SCATTERING MATRIX. Kennaugh considered powers received from polarized waves scattered by stable radar targets. Only backscattering has been described in his Reports, and it is a pity that his '*unpublished notes of the year 1980*', extending his concepts to the case of bistatic scattering or nonsymmetric matrices, mentioned in [1], are not available in the literature.

Kennaugh's results determine '*the variation of back-scattered RCS with common transmit-receive polarization P given by a simple geometrical construction on the Poincaré sphere*'. Altogether *three constructions* have been proposed when plotting *P* and the target null-polarizations N_1 and N_2 on the Poincaré sphere of *unit diameter*. The factor of proportionality, called the effective crossection (ECS), has been introduced and defined by the squared sum of square roots of RCS for points *M* and *E*, with *M* representing polarization of maximum RCS, and with *E* being antipodal point versus *M*. It equals also the trace of the symmetric scattering matrix when presented in the real diagonal form:

$$\mathrm{ECS} = \left(\sqrt{\mathrm{RCS}(M)} + \sqrt{\mathrm{RCS}(E)}\right)^2 = (a_{11} + a_{22})^2 \qquad (1)$$

Additionally, the *C* point has been determined as the center of the N_1N_2 chord located on the *ME* diameter perpendicular to the N_1N_2 chord with distances $MC \geq EC$.

Two of those three constructions present powers (for the unit transmit/receive antenna heights or unit Stokes four-vectors):

$$P_{\text{rec co-pol}} \equiv \mathrm{RCS}_{\text{co-pol}}(P) = \mathrm{ECS} \cdot \left[(PN_1) \cdot (PN_2)\right]^2 \qquad (2)$$

$$P_{\text{scatt}} \equiv \mathrm{RCS}(P) = \mathrm{RCS}_{\text{co-pol}}(P) + \mathrm{RCS}_{\text{cross-pol}}(P) = \mathrm{ECS} \cdot (PC)^2 \qquad (3)$$

The third construction '*determines the polarization of the reflected wave by first locating the end point of the chord from P through C, then rotating this point by* 180^0 *about a diameter parallel to the chord* N_1N_2.'

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There is no doubt about the fundamental practical significance of all those three constructions.

The first one enables one, by just a glance at the Poincaré sphere with two null-polarization points, to mentally trace on it the curves of constant RCS. Owing to that, for example, the possibility of immediate determination of the allowed areas of transmit-receive polarizations for the required cancellation of radar clutter is noteworthy and beyond discussion.

The second construction allows for visual presentation of the level of the total power scattered by the radar target. It clearly indicates the significance, discovered by Kennaugh, of the inversion point C determining the polarization dependence of the scattered power.

The third construction explains another significance of the inversion C point concept. It allows for immediate determination of the scattered polarization by two geometric operations on the *Poincaré sphere model* of the scattering matrix. Those subsequent operations, called '*inversion*' and '*rotation after inversion*', in the case of the symmetric scattering matrix uniquely depend on location of two null-polarization points, determining both the inversion point and the axis of rotation after inversion. That agrees with the total number of 6 real parameters of the symmetric scattering matrix. Its 4 parameters can be considered as two pairs of spherical coordinates of the two null-polarization points. The fifth parameter will be expressed by the ECS factor, and the sixth one will determine the general phase of the matrix, being of no influence on the received power.

EXTENSION OF THE KENNAUGH'S CONCEPTS TO THE CASE OF NONSYMMETRICAL MATRICES [4], [5]. The nonsymmetric matrix can be decomposed into the sum of its symmetric and nonsymmetric parts. For bistatic scattering with the same transmit and receive polarization the symmetric part only of the matrix should be taken into account because its nonsymmetric part behaves like an orthogonalizer and does not contribute to forming of the scattered wave. Therefore, for the co-pol channel, the first Kennaugh's geometric construction remains unchanged with the only exception that the ECS value corresponds to the symmetric part only of the scattering matrix. For different transmit and receive polarizations, the ECS, being used for the next two constructions, should be computed as the span of the nonsymmetric matrix plus doubled absolute value of its determinant.

An extension of the theory to nonsymmetric matrices requires introduction of two more real parameters of the scattering matrix. It appears that the inversion point I deviates from the center C of the N_1N_2 chord, retaining however its fundamental property of determination of the scattered power. Location of that point can now be expressed by three rectangular coordinates in the so called characteristic coordinate system (CCS) of Stokes parameters which can be obtained from the original system of traditional Stokes parameters by its rotation, for example, by three Euler angles. With the ECS factor and the general phase, altogether 8 real parameters of the nonsymmetric scattering matrix are thus determined.

The Poincaré sphere model of the nonsymmetric matrix differs from that of the symmetric matrix by something more than by different location of the inversion point versus the null-polarizations. For the symmetric matrix the M point represents not only the polarization of maximum scattered power but also the so-called eigenpolarization (the 'cross-pol null' or the 'attracting' E_2 polarization) and the transmit-receive antenna polarization corresponding to the maximum received power (the 'co-pol max' or the 'characteristic' K polarization). For nonsymmetric matrices the three polarizations, M, E_2 and K, are separated, and K only remains at the end of the diameter perpendicular to the N_1N_2 chord. The axis of rotation after inversion may become not parallel to the N_1N_2 chord, nevertheless it remains in the plane parallel to that chord. Also the angle of rotation after inversion may differ from 180^0. It is an interesting feature of the model that the axis and angle of rotation, as well as all special polarization points (N_1, N_2, M, E_2, K, and all others), can be found for given coordinates of the inversion I point in the CCS (two solutions exist for some locations of the I point [4], [5]).

MATRIX AND QUATERNIONIC REPRESENTATIONS OF THE INVERSION AND ROTATION [7]. The amplitude bistatic scattering equation expressed in terms of halves of the Poincaré sphere angles of polarization tangential phasors P and S (of the incident and scattered waves), with column vectors expressed in terms of their analytic parameters, is an extension of Kennaugh's formulation and can be presented in the form

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \cos\gamma \exp[-j(\delta+\varepsilon)] \\ \sin\gamma \exp[j(\delta-\varepsilon)] \end{bmatrix}^P = \sqrt{\mathrm{RCS}(P)} \begin{bmatrix} \cos\gamma \exp[j(\delta+\varepsilon)] \\ \sin\gamma \exp[-j(\delta-\varepsilon)] \end{bmatrix}^S \tag{4}$$

The Sinclair scattering matrix $\boldsymbol{A}$ here applied can be decomposed into product of the inversion and rotation matrices with the general phase term and the ECS factor as follows:

$$\boldsymbol{A} = \exp(j\xi) \cdot \sqrt{\mathrm{ECS}} \cdot (C^{ROT}*) A^{INV} \tag{5}$$

with

$$2\xi = \arg\det \boldsymbol{A}, \qquad \mathrm{ECS} = Span\boldsymbol{A} + 2\,|\det \boldsymbol{A}| \tag{6}$$

$$C^{ROT}* = \begin{bmatrix} \cos\phi + jn_1 \sin\phi & (-n_3 + jn_2)\sin\phi \\ (n_3 + jn_2)\sin\phi & \cos\phi - jn_1\sin\phi \end{bmatrix} \tag{7}$$

$$A^{INV} = \frac{1}{\mathrm{ECS}} \begin{bmatrix} \mathrm{b}_3 + j\mathrm{b}_5 & \dfrac{\mathrm{ECS}}{2} + \mathrm{b}_1 \\ -\dfrac{\mathrm{ECS}}{2} - \mathrm{b}_1 & -\mathrm{b}_3 + j\mathrm{b}_5 \end{bmatrix} = \begin{bmatrix} -\mathrm{U} - j\mathrm{V} & \frac{1}{2} + \mathrm{Q} \\ -\frac{1}{2} + \mathrm{Q} & \mathrm{U} - j\mathrm{V} \end{bmatrix} \tag{8}$$

where:

$$\begin{bmatrix} \cos\phi \\ n_1 \sin\phi \\ n_2 \sin\phi \\ n_3 \sin\phi \end{bmatrix} = \frac{1}{\sqrt{\mathrm{ECS}}} \begin{bmatrix} \mathrm{Re}\{(a_{12} - a_{21})\exp(-j\xi)\} \\ \mathrm{Im}\{(a_{12} + a_{21})\exp(-j\xi)\} \\ \mathrm{Im}\{(a_{22} - a_{11})\exp(-j\xi)\} \\ \mathrm{Re}\{(a_{22} + a_{11})\exp(-j\xi)\} \end{bmatrix}, \quad \begin{bmatrix} \mathrm{Q} \\ \mathrm{U} \\ \mathrm{V} \end{bmatrix} = \frac{-1}{\mathrm{ECS}} \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_3 \\ \mathrm{b}_5 \end{bmatrix} \tag{9a,b}$$

$2\mathrm{b}_1 = a_{11}a_{11}* - a_{12}a_{12}* + a_{21}a_{21}* - a_{22}a_{22}*$, $\mathrm{b}_3 + j\mathrm{b}_5 = a_{11}a_{12}* + a_{21}a_{22}*$,
Q, U and V are rectangular coordinates of the inversion I point inside the Poincaré sphere of unit diameter, and n_1, n_2, n_3 are directional cosines of the axis of rotation by the 2ϕ angle.

For the pure quaternion units e_1, e_2, e_3 and their multiplication rules:

$e_k^2 = -1,\ \ k = 1,2,3;\ \ \ e_1 e_2 = e_3 = -e_2 e_1,\ \ \ e_2 e_3 = e_1 = -e_3 e_2,\ \ \ e_3 e_1 = e_2 = -e_1 e_3$

the power scattering equation can be presented as ([6], [7]):

$$(\mathrm{ECS}/4) \cdot \{\mathcal{C}^{ROT}(\mathcal{A}^{INV}*)(\mathcal{P}^{P}*)\tilde{\mathcal{A}}^{INV}\tilde{\mathcal{C}}^{ROT}*\} = \mathrm{RCS}(P) \cdot \mathcal{P}^{S} \tag{10}$$

with (11)

$$\mathcal{P}^{S} = 1 + j2(\mathrm{q}^S e_1 + \mathrm{u}^S e_2 + \mathrm{v}^S e_3),\quad \mathcal{P}^{P}* = 1 + j2(\mathrm{q}^P e_1 + \mathrm{u}^P e_2 - \mathrm{v}^P e_3)$$

$$\mathcal{A}^{INV}* = -e_3\{1 - j2(\mathrm{Q}e_1 + \mathrm{U}e_2 - \mathrm{V}e_3)\},\quad \tilde{\mathcal{A}}^{INV} = \{1 - j2(\mathrm{Q}e_1 + \mathrm{U}e_2 - \mathrm{V}e_3)\}e_3$$

$$\begin{aligned} \mathcal{C}^{ROT} &= \cos\phi + (n_1 e_1 + n_2 e_2 + n_3 e_3)\sin\phi \\ \tilde{\mathcal{C}}^{ROT}* &= \cos\phi - (n_1 e_1 + n_2 e_2 + n_3 e_3)\sin\phi \end{aligned} \tag{12}$$

where q, u, v are rectangular coordinates of the polarization point on the Poincaré sphere of *unit diameter*, with

$$\begin{bmatrix} \mathrm{q} \\ \mathrm{u} \\ \mathrm{v} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \cos 2\gamma \\ \sin 2\gamma \cos 2\delta \\ \sin 2\gamma \sin 2\delta \end{bmatrix};\quad \mathrm{q}^2 + \mathrm{u}^2 + \mathrm{v}^2 = 1/4 \tag{13}$$

where Q, U, V are rectangular coordinates of the inversion point inside the Poincaré sphere of unit diameter, and where

$$\mathrm{RCS}(P) = \tfrac{1}{2}\, Span\boldsymbol{A} + 2(\mathrm{b}_1 \mathrm{q}^P + \mathrm{b}_3 \mathrm{u}^P + \mathrm{b}_5 \mathrm{v}^P) \tag{14}$$

The power received from the scattered wave for the unit incident power can be presented as the RCS(P) multiplied by the square of the cosine of half an angle on the Poincare sphere between polarization points R and S, of the receiving antenna and the scattered wave, or by the dot product $\bullet$ of the corresponding unit quaternions defined as follows:

$$P_r = \mathrm{RCS}(P) \cdot \tfrac{1}{2}(\mathcal{P}^{R} \bullet \mathcal{P}^{S}) = \mathrm{RCS}(P) \cdot \tfrac{1}{2}\{1 + 4(\mathrm{q}^R \mathrm{q}^S + \mathrm{u}^R \mathrm{u}^S + \mathrm{v}^R \mathrm{v}^S)\} \tag{15}$$

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