Kennaugh and the Dual Space Approach to Radar Polarimetry

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Introduction

This contribution emphasizes the role of the voltage equation in the early development of backscatter radar polarimetry as a bilinear form as pursued by E.M. Kennaugh and its relation to the time reversal operation.

1. Historical remarks to Kennaugh's dual space approach

At the middle of the last century the technological concept of radar and the physical concept of optical polarimetry had established themselves as well proven engineering and scientific research areas. However, the mixture of both concepts applied to the case of most interest for military and civilian purposes, i.e. backscatter radar polarimetry, at first presented unexpected theoretical and practical difficulties and misunderstandings that in part prevail up to the present times and offers hidden pitfalls to the unwary. For a recent review of radar polarimetry and its applications, see Boerner et al [2].

The reason for this uneasiness should be clear, however. Backscatter polarimetry can be described correctly only using the concept of 'time inversion' and the associated 'complex conjugation' operation to the electromagnetic polarization descriptors (Jones or Stokes vectors). This is obviously a nonlinear (antilinear) operator, well-known in quantum mechanics but quite infamiliar to engineering sciences, see Lüneburg [3]. Related to to this new concept, there is additionally a re-definition of the state of polarization describing plane electromagnetic waves travelling in opposite directions. Every attempt to avoid the use of such a nonlinear operation by passage to another coordinate system or introducing new conventions either turned out to be inconsistent or failed at some later stage. In this respect the IEEE Standard Test Procedures and Definitions of Terms for Antennas [4] are incomplete, inconsistent and contradictory if not incorrect.

It was Professor Edward Morton Kennaugh (1922-1983) at the Ohio ElectroScience Laboratory who suggested and extensively used a correct alternative way to describe characteristics of radar polarimetry and to solve this conundrum. Kennaugh's approach even avoids any conflicts with the IEEE Standards [4]. Referring to these standards a transmitted wave $\vec{E}^{\,t}$ and a reflected (backscattered) wave $\vec{E}^{\,r}$ represented by the same components in a common linear coordinate system will differ not only in the direction of propagation but *will actually have polarization ellipses which are inclined at equal and opposite angles to the common reference axis* (refer to Kennaugh [5], Fig. 2). Being interested mainly in optimal characteristic polarization states Kennaugh avoids to speak of the polarization of the reflected wave in general but refers to the definition of antenna states for for transmission and reception in a common linear polarization basis. If the vector height of an elliptically polarized antenna is given by $\vec{h}$ (transmission), the elliptically polarized incident wave that is best receives would be given by $\vec{h}^*$, where the asterisk denotes complex conjugation. This operation describes the time reversal. We may state, however, the unfortunate side effect that in the following decades radar polarimetry sometimes appeared as a clever hodgepodge of wave theory and antenna network performance (see Hubbert [6]), an opinion that to our believe and knowledge was never supported by Kennaugh. He was a pragmatic mathematical engineer working in antenna

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theory and electrodynamics who had developed over the years a deeply rooted intuition how practical problems in radar polarimetry could not only be formulated but effectively be solved down to numerical results with advantageous illustrative representations. In particular we refer to his M.Sc. Thesis [5]. Kennaugh's work was later collected and systematically extended by Huynen [7]. Huynen coined the term 'polarization fork' on the Poincareé sphere for characterizing optimal polarization states; a concept that earlier on had been discovered in Kennaugh's thesis [5].

2. Time reversal and the voltage equation

In radar backscattering the Jones vectors as polarization descriptors must be endowed with a 'tag', indicating the direction of propagation. Jones vectors with different tags belong to different (conjugate) linear propagation vector spaces. The passage from one of these spaces to the other can be accomplished by time inversion and corresponds in a common linear polarization basis to complex conjugation

$$\vec{E}_+ \quad \Longrightarrow \quad \vec{E}_- = \vec{E}_+^* \quad \text{(nonlinear operation; common linear polarization basis).}$$

The Jones vectors $\vec{E}_-$ and $\vec{E}_+^*$ describe waves travelling in opposite directions but having by definition the same state of polarization. This includes and supersedes the definition of states of polarization of antennas for transmission and reception, see Sinclair [9], Rumsey [10] and Graves [11].

Let W be a linear vector space over the field $\mathcal{C}$. Then its dual space W' is the set of all linear functionals from $W \to \mathcal{C}$; this is also a $\mathcal{C}$-vector space. If $\{\vec{w}_1, \vec{w}_2, \cdots, \vec{w}_n\}$ is a basis for W, the elements $\{\vec{w'}_1, \vec{w'}_2, \cdots, \vec{w'}_n\} \subseteqq W'$ satisfy the relations $\vec{w'}_i(\vec{w}_j) = \delta_{ij}$ for all i and j. For a finite-dimensional linear vector space W and W' have the same dimension. In radar polarimetry the role of the space that is dual to the incident Jones vector space is given by the voltage equation or scalar radar brightness

$$V \equiv V(\vec{h}_r, \vec{h}_t) = (\vec{h}_r, S\vec{h}_t) \equiv \vec{h}_r^T S \vec{h}_t$$

where $\vec{h}_t$ and $\vec{h}_r$ are the (normalized) 2-component Jones vectors of the incident/transmit and receiving antenna, respectively, and S is the (relative) Sinclair scattering matrix All quantities are assumed to be expressed in a common linear polarization basis for the domain and the range of the Sinclair matrix. Due to reciprocity the Sinclair backscatter matrices for point targets are symmetrical and have five independent parameters, see Kennaugh [5], Mott [12]. Traditionally since the pioneering investigations by Kennaugh and others and later on by Huynen the voltage equation has been the primary object for extracting information about the scattering target rather than the electromagnetic fields themselves. But rather than treating the voltage equation as a bilinear form by itself the traditional approach consists in splitting up the voltage equation into two parts: the scattering effects of the target produced by an incident wave and the effect of the receiver antenna. This point of view may have contributed to the incorrect, unfortunate and misleading impression that radar polarimetry is a clever mixture of electromagnetic scattering theory and radar network performance leading to formidable misinterpretations in the literature, see Lüneburg [8]. The squared absolute value $P = |V|^2$ is known as power transfer, echoing are or "radar brightness".

Mathematically the voltage equation V for backscattering is a symmetric bilinear form, i.e., a functional $B : WxW \to \mathcal{C}$, that is linear in either variable if the other is fixed, cf Grove [13]. Here the Sinclair matrix S appears as the matrix representation of the bilinear voltage form with respect to a linear polarization basis. The use of a particular basis leads to a particular matrix representation $\hat{B}$ of the bilinear form B. This contribution will present some results of the theory of bilinear forms and the associated quadratic forms (co-polar voltages)

$$Q(\vec{w}) \equiv B(\vec{w}, \vec{w}) \quad \Longleftrightarrow \quad B(\vec{v}, \vec{w}) = \frac{1}{2}\left[Q(\vec{v} + \vec{w}) - Q(\vec{v}) - Q(\vec{w})\right]$$

as far as they are relevant to radar polarimetry. Of special importance for quadratic forms are the isotropic vectors and their corresponding subspaces that coincide with the co-polar nulls of backscatter radar polarimetry and form a 2-dimensional hyperbolic plane. An unitary change of polarization basis U implies an unitary congruence or unitary consimilarity transformation of the matrix representation of B:

$$\hat{B} \to \hat{B}' = U^T \hat{B} U \quad \text{(unitary consimilarity)}.$$

This is the well-known transformation rule for the basis transformation of the backscatter Sinclair matrix whose validity is often questioned since the correct interpretation in terms of (directed) Jones vectors requires the acceptance of the time reversal operation, see Lüneburg [3]. This treansformation behavior for backscattering should be contrasted with that for forward scattering (transmission) with a Jones matrix $\hat{J}$ replacing the Sinclair matrix $\hat{S}$

$$\hat{J} \to \hat{J}' = U^\dagger \hat{J} U \quad \text{(unitary similarity)}$$

where $U^\dagger U = UU^\dagger = I$. If the domain and range of the Sinclair matrix are expanded in nonlinear basis vectors (for instance left/right circular polarization states) then also the basis vectors themselves change under the time reversal operation; this situation is fully described in Lüneburg [14].

The preceding formulation of backscatter radar polarimetry is completely equivalent to the alternative formulation in terms of directed Jones vectors but avoids the explicit use of the time reversal operation. Kennaugh was able to derive the so-called optimal states of polarization which includes in particular co-polar maxima (cross-polar nulls), co-polar nulls, and cross-polar maxima. These results are elementarily obtained by the method of Lagrange multiplies (to take into account constraints) combined with parameter estimations. Working in the original Jones vector space the derivation of these optimal states of polarization leads to problems of the existence and uniqueness os solutions of coneigenvector/coneigenvalue equations (Kennaugh's pseudo eigenvalue equations) or the unitary condiagonalization of complex symmetric matrices. This approach has been pursued in part by Huynen [7] and more recently by Lüneburg [3], see also Boerner et al [2].

3. Conclusions and outlook

The passage from the linear vector space W to the dual linear vector space W' with the corresponding linear form, i.e., the voltage equation V, used extensively by Kennaugh and others, brought radar polarimetry in contact with branches of rather esoteric pure mathematics, namely the geometry of the classical groups and geometric algebra, cf Grove [13] or Taylor [15]. This theory was prospering already during the entire last century with numerous interrelations to other fields of mathematics, like group theory, geometry and analytic algebra. Many theorems that were considered as new in radar polarimetry actually have a long and important history in their special mathematical context. For instance, the derivation of the co-pol maxima (cross-pol nulls) by unitary condiagonalization goes back as far as to L. Autonne [1] in 1915 and has been rediscovered several times later on, and the theory of co-pol nulls or isotropic vectors is well known in geometric algebra and is related to Witt's theorem [15]. Many new discoveries in geometric algebra (Tits buildings, flags and apartments, see Taylor [15]) are waiting to be related to the theory of polarimetry as well as to ellipsometry in optics.

These remarks do not diminish the achievements and progresses made by Kennaugh in the pioneering stages of radar polarimetry but honor him. He was the first to clear the undergrowth that covered and surrounded early radar polarimetry and opened the door to other points of view that involve interesting and by far not yet fully exploited branches of mathematics. This refers to the application of Clifford algebra, quaternions, 2- and 4-component spinor theory and in general to the geometrization and visulaization of polarimetric concepts. These aspects will be covered at least in part by other contributions to this special session in honor of Edward Morton Kennaugh.

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