# Metamaterials Composed of Fractal Sphere Molecules

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## Abstract

The properties of artificial dielectric metamaterials depend primarily on the type of inclusions or “molecules” considered as well as their distribution in a host medium. Some common types of molecule geometries that were considered in the past for artificial dielectric materials include simple dipoles and spheres. In this paper, we consider a fractal sphere approach, combining aspects of both the dipole and sphere resulting in an inclusion with a greater number of resonances and resonant behavior at lower frequencies than either their dipole or sphere counterparts of the same size. The results of this investigation suggest that the fractal sphere molecules allow us to push the lower frequency band and in some instances also provide multiband response.

## I. Introduction

The development of artificial dielectric metamaterials in recent years has led to advancements in a variety of areas including antennas, waveguides, polarizers, and electromagnetic absorbing materials. These artificial dielectric metamaterials are formed by introducing inclusions (i.e., molecules) into a background dielectric material. These inclusions change the electromagnetic properties of the dielectric material of interest. Metamaterials will be considered here that are composed of a new type of molecule called a fractal sphere. A fractal sphere is a collection of symmetric self-similar fractal tree dipoles arranged to form a sphere-like structure as illustrated in Figure 1 [1].

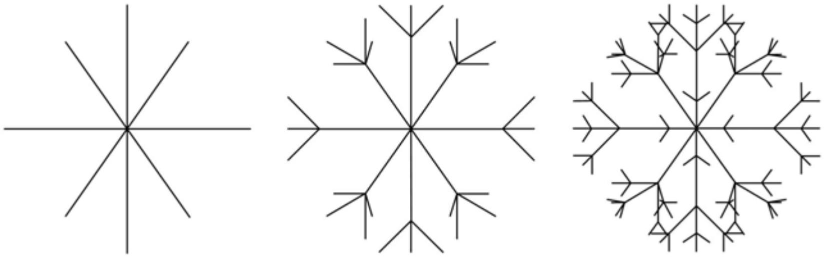

**Figure 1.** Fractal sphere geometry as viewed along any axis for each of the first three iterations.

The behavior of the backscatter cross-section versus frequency has been compared in [1] for the first three iterations of the fractal sphere shown in Figure 1. The results of this comparison demonstrate the multiband properties associated with fractal spheres, where the number of resonances or bands are seen to increase with each successive iteration. Another trend observed in [1] is the first resonance shifts down to a lower frequency each time a new stage of growth is added to the fractal sphere. A numerical comparison of the size reduction of the fractal sphere over the solid sphere is provided in Table 1. The table is compiled by designing a solid sphere, regardless of radius, whose first resonance aligns with the first resonance of a first, second and third iteration fractal sphere.

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| Iteration | Radius of Fractal Sphere (mm) | Radius of Solid Sphere (mm) | Percent Size Increase (%) |
|---|---|---|---|
| 1 | 2.50 | 2.50 | 0.00 |
| 2 | 3.49 | 5.00 | 43.27 |
| 3 | 3.67 | 7.50 | 104.36 |

**Table 1.** Size comparison of a solid sphere to an equivalent fractal sphere for the first three iterations.

## II. Multipole Approximation

A multipole approximation [2,3] is developed here for the scattered electric field from an individual fractal sphere. This represents an important step towards determining the effective medium parameters associated with an artificial dielectric metamaterial composed of these fractal sphere molecules. Suppose that the fractal sphere is divided into a sequence of segments, each of which is assumed to be an ideal dipole. Hence, the total scattered field produced by an arbitrary segment of the fractal sphere centered at the point ($x_0$, $y_0$, $z_0$) may be represented by

$$\mathbf{E}_T(r) = e^{-jk\left(x_0 \sin\theta\cos\phi + y_0 \sin\theta \sin\phi + z_0 \cos\theta\right)} \mathbf{E}_{sc}(r) \tag{1}$$

where

$$\mathbf{E}_{sc}(r) = \frac{k^2}{4\pi\varepsilon_0}\frac{e^{-jkr}}{r}\left[(\hat{\mathbf{r}}\times\mathbf{p})\times\hat{\mathbf{r}} - (\hat{\mathbf{r}}\times\mathbf{m})\right] \tag{2}$$

and where the electric dipole moment **p** and the magnetic dipole moment **m** are given by

$$\mathbf{p} = \frac{1}{j\omega}\int_C \mathbf{J}\, ds' \tag{3}$$

$$\mathbf{m} = \frac{k}{2\omega}\int_C (\mathbf{r}'\times\mathbf{J})\, ds' \tag{4}$$

Figure 2 shows a comparison of the resultant multipole approximation with rigorous Method of Moments (MoM) simulations for the backscatter cross-section of a third iteration fractal sphere.

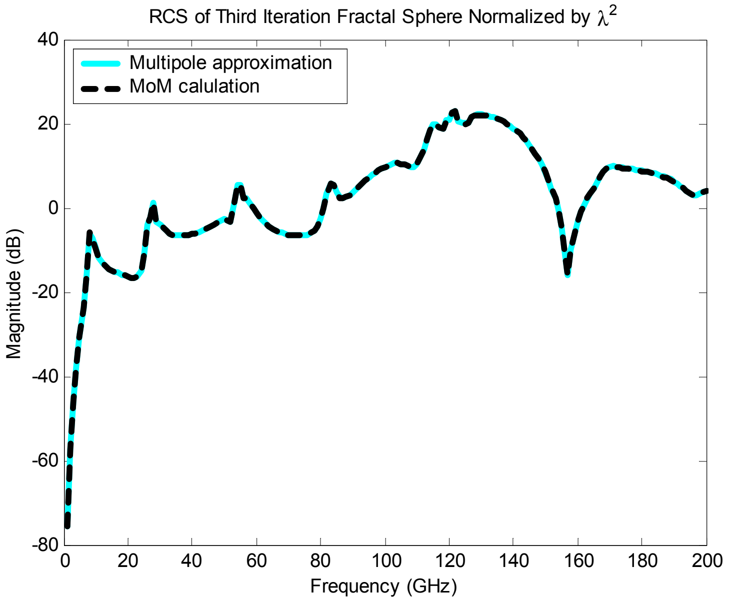

**Figure 2.** Multipole approximation versus MoM calculation for a third iteration fractal sphere.

## III. Effective Dielectric Constant

The effective polarizabilities must first be computed before evaluating the effective dielectric constant of the artificial dielectric metamaterial [3,4]. The polarizabilities are dyadic quantities that may be extracted from the following set of equations:

$$\mathbf{p} = \varepsilon(\overline{\overline{a}}_{ee}\mathbf{E} + \overline{\overline{a}}_{em}\eta\mathbf{H}) \tag{5}$$

$$\mathbf{m} = \frac{\overline{\overline{a}}_{me}}{\eta}\mathbf{E} + \overline{\overline{a}}_{mm}\mathbf{H} \tag{6}$$

We examine the dipole moments for 6 different incident plane waves along the positive and negative directions of each axis. Once these values are obtained, they are appropriately averaged and substituted into (7) to calculate the effective dielectric constant of the metamaterial.

$$\varepsilon_{eff} = \varepsilon \frac{\left(1 + 2N\frac{a_{ee}}{3}\right)\left(1 - N\frac{a_{mm}}{3}\right) - 2N^2\frac{a_{em}^2}{9}}{\left(1 - N\frac{a_{ee}}{3}\right)\left(1 - N\frac{a_{mm}}{3}\right) + N^2\frac{a_{em}^2}{9}} \tag{7}$$

Along with the polarizabilities, the inclusion volume density must also be specified in (7). In this case, we consider the maximum number of inclusions that can be placed in a cubic meter volume without intersection. The resultant real and imaginary parts of the effective dielectric constant for each iteration of the fractal sphere are shown in Figures 3 and 4 respectively. Also shown in Figures 3 and 4 for comparison purposes are plots of the effective dielectric constant for an artificial dielectric metamaterial consisting of conventional dipole inclusions. Figures 3 and 4 demonstrate that each successive iteration of the fractal sphere molecules produces a characteristic downward shift in the corresponding peak value of the effective dielectric constant. Also, it is seen that a medium composed of third iteration fractal spheres behaves as a dualband artificial dielectric metamaterial.

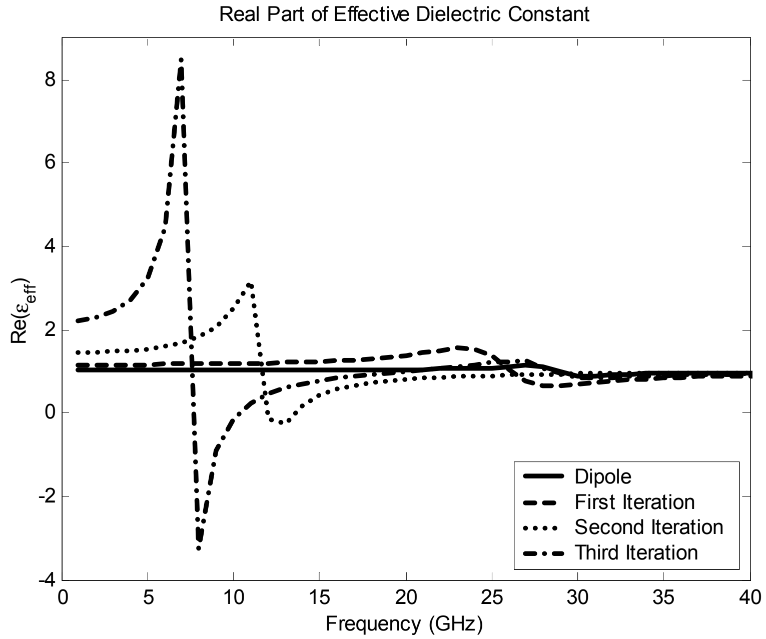

**Figure 3.** Real part of the effective dielectric constant for $N_{MAX}$.

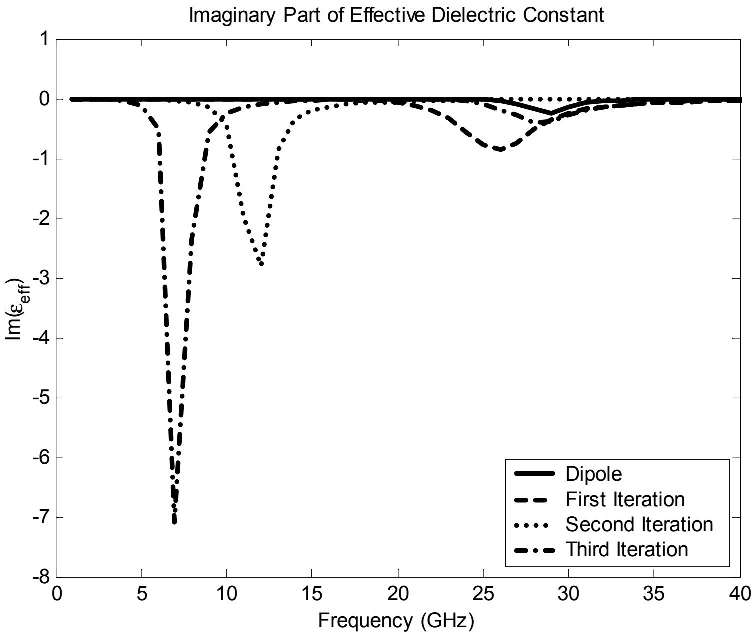

**Figure 4.** Imaginary part of the effective dielectric constant for $N_{MAX}$.

## IV. Conclusions

It has recently been demonstrated that fractal spheres possess backscattering properties that are characteristic of both linear dipoles and solid spheres. It was also found that the self-similar geometrical structure of these fractal spheres produced a characteristic multiband response in their backscattering cross-section. Moreover, it has been shown that fractal spheres have a lower resonant frequency than solid spheres of the same physical size. These properties have been exploited in this paper to develop a new class of low-frequency multiband artificial dielectric metamaterials.

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## References

[1] M. Facchine, and D. H. Werner, "Electromagnetic Scattering from Fractal Spheres," *2002 IEEE AP-S International Symposium,* June 2002, Vol. 3, pp. 106-109.

[2] J. D. Jackson, *Classical Electrodynamics*, John Wiley & Sons, Inc., New York, NY, 1966.

[3] D. H. Werner, and R. Mittra, *Frontiers in Electromagnetics*, IEEE Press, Piscataway, NJ, 2000.

[4] A. Sihvola, *Electromagnetic Mixing Formulas and Applications*, The Institute of Electrical Engineers, London, United Kingdom, 1999.