

# Measurement of Circuit-Embedded Artificial Permeability Meta-Materials Utilizing Frequency Extended Perturbation Method

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While low-loss dielectric materials offer a wide range of permittivity values, the availability of magnetic materials for microwave applications is woefully limited. Advanced artificial meta-materials which provide desirable bulk EM characteristics not available in naturally occurring materials have been developed, presenting a challenge to existing measurement methods. Such materials promise among other things, a wide range of selectable possible permeability values in the microwave region at low losses. Recently developed meta-materials provide highly desirable bulk properties including reactive impedance surfaces yielding greatly improved miniaturized patched antennas and embedded-circuit resonators yielding engineered selectable anisotropic or isotropic permittivity and permeability.

Potential applications of embedded-circuit resonators are numerous and promising, but before application, such materials must be fabricated and characterized to validate their theoretical permittivity and permeability. In order to measure these samples across a bandwidth a novel method for determining  $\epsilon_r$  and  $\mu_r$  as a function of frequency from a single resonant method is presented. This technique has the benefit of high accuracy at the resonant frequency while simultaneously providing results across a bandwidth. This is particularly important for a dispersive medium such as this meta-material.

## **Embedded-Circuit resonators for Engineered Permittivity and Permeability**

“Metamaterials” are engineered (nano) composites that exhibit superior properties that are not found in nature and not observed in the constituent materials. By embedding circuit elements which are very small relative to the operating wavelength, the electric or magnetic energy storage property of the circuit may be imparted to the larger bulk material. In natural materials atoms and molecules perform the role of electric and magnetic energy storage to determine the materials permittivity and permeability. In metamaterials, electromagnetically small circuits and engineered inclusions may be employed for this same purpose.

One of the simplest circuits to demonstrate this phenomenon is an LC resonator in a natural dielectric medium. Since the electric and magnetic storage of the embedded circuits are a consequence of the geometry of the microstrip elements, the bulk permittivity and permeability of this manufactured medium are independently controllable by adjusting the microstrip geometry.

In planar array medium of such elements the permittivity ( $\epsilon$ ) and permeability ( $\mu$ ) exhibit anisotropic behavior such that the effective permeability and permittivity tensor are given by  $\mu_y = \mu_{eff}$ ,  $\mu_x = \mu_z = \mu_0$  and  $\epsilon_z = \epsilon_{eff}$  while  $\epsilon_x = \epsilon_y = \epsilon_0$ . Analytical formulations of  $\epsilon_{eff}$  and  $\mu_{eff}$  as a function circuit geometry exist, but only experimentation can validate them. (K. Sarabandi, “Electro-Ferromagnetic Tunable Permeability, Band-Gap, and Bi-Anisotropic Meta-Materials Utilizing Embedded-Circuits” AP Symposium, 2003).

An engineered Meta-Material composed of embedded circuits to achieve artificial permeability is designed, fabricated, and measured.

## **Frequency Extended Perturbation Method**

Traditional perturbation method is applied to determine  $\mu_r$  or  $\epsilon_r$  with high accuracy at a single frequency. By placing a small sample in the null of the magnetic field of a resonant cavity and observing the resonant frequency shift  $\epsilon_r$  can be determined. At the E-field null  $\mu_r$  can be measured.

In a more generally conception every point in the perturbed system responses frequency response is a point-to-point mapping of the unperturbed system response. The traditional perturbation method equations represent the frequency portion of the mapping algorithm applied at the resonant peak only. In the frequency extended perturbation method, frequency mapping is applied to determine  $\mu_r$  or  $\epsilon_r$  across a bandwidth. To develop the magnitude mapping algorithm the sample losses and the compression/expansion of the spectral response must be accounted for as they will tend to narrow or widen the 3-dB bandwidth while raising/lowering the peak transmission coefficient. Luckily, if the change in ‘Q’ between perturbed and unperturbed cases and dispersion are minor, this difference can be ignored and the predicted system response easily obtained.

To validate this inversion method a small volume fraction (1%) non-dispersive dielectric of  $\epsilon_r=2.17$  with low loss ( $\text{Tan}\delta=0.001$ ) was placed in a physical cavity and the perturbed and unperturbed system responses were measured, results validates the extended perturbation method is +/- 10% accurate over a 15% bandwidth.